Lesson 7 New Keynesian Model (2)

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14.3.3 Equilibrium

- Market clearing condition is given by:

$$y_t(j) = C_t(j) \quad (14.28)$$

- Similar to aggregated consumption Eq.(14.2), aggregated output is given by:

$$Y_t \equiv \left[\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (14.29)$$

where Y_t denotes the aggregated output, namely, GDP.

- Similar to Eq. (14.8) derived from Eq.(14.2), we have the demand schedule from Eq. (14.29) as follows:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} Y_t \quad (14.30)$$

- Plugging Eqs.(14.8) and (14.30) into (14.28) yields:

$$Y_t = C_t \quad (14.31)$$

which is an equilibrium condition same as it in classical monetary model. Similar to classical monetary model, the GDP definitely equals to the consumption because we ignore capital, foreign and government and expenditure is just consumption.

- Needless to say, log-linearizing Eq.(14.31) yields:

$$y_t = c_t \quad (14.32)$$

- Plugging Eq.(14.32) into Eq.(14.12) yields:

$$y_t = \mathrm{E}_t(y_{t+1}) - \hat{i}_t + \mathrm{E}_t(\pi_{t+1}) \quad (14.33)$$

- Next, we consider prodiction function. Plugging production function Eq.(14.14) and the demand schdule Eq.(14.30) into the equilibrium condition on the labor market yields:

$$\begin{aligned} N_t &\equiv \int_0^1 N_t(j)dj \\ &= \int_0^1 Y_t(j)A_t^{-1}dj \\ &= Z_t Y_t A_t^{-1} \end{aligned} \quad (14.34)$$

where $Z_t \equiv \int_0^1 \left(P_t(j)/P_t\right)^{-\varepsilon} dj$ denotes the Price Dispersion. P_t is an average of prices of any good j $P_t(j)$ and total amount of goods is 1. Thus, P_t equals to value raised an average of price dispersion to $-\varepsilon$.

- Although $Z_t = 1$, the order is 2. Thus, Z_t is ignored on log-linearization.
- Thus, log-linearizing Eq.(14.34) yields:

$$y_t = a_t + n_t \quad (14.35)$$

which is same as one in classical monetary model when $\alpha = 1$, Eq.(3.22).

- Now we consider the marginal cost. Marginal cost is additional cost when output increases one unit, that is:

$$MC_t^n = \frac{W_t(1-\tau)\partial N_t}{\partial Y_t}$$

where τ denotes the employment subsidy.

- When we ignore the employment subsidy τ, the numerator is additional cost because the nominal wage W_t is unit cost and ∂N_t is additional employment. The denominator is additional output and the RHS is an additional cost when output increases one unit.

- This can be rewritten as:

$$MC_t^n = \frac{W_t(1-\tau)}{\partial Y_t / \partial N_t}$$

- The numerator in the RHS is unit cost while the denominator is the marginal product of labor which is additional output when employment increases one unit.
- Here, we calculate $\partial Y_t/\partial N_t$. By paying attention to Z_t=1, Eq.(14.34) can be rewritten as:

$$Y_t = A_t N_t$$

- Then, the marginal product of labor is $\partial Y_t/\partial N_t = A_t$. Thus, the nominal margina cost is given by:

$$MC_t^n = \frac{W_t(1-\tau)}{A_t}$$

- Plugging the definition of the real marginal cost $MC_t \equiv MC_t^n / P_t$ into this equality, we have:

$$MC_t = \frac{W_t(1-\tau)}{P_t A_t}$$

- Plugging Eq.(14.11) into this yields:

$$MC_t = \frac{N_t C_t (1-\tau)}{A_t} \qquad (14.36)$$

- By ignoring the employment subsidy τ and the productivity A_t Eq.(14.36) shows that the real marginal cost equals to the employment times the consumption.

- An increase in the employment (an increase in the marginal disutility of labor) increases the real marginal cost through the real wage.
- An increase in consumption (A decrease in the marginal utility of consumption) increases the real marginal cost through an increase in the real wage.
- The increase in the employment subsidy and the increase in the productivity decreases the real marginal cost.

- Log-linearizing $MC_t = \frac{W_t(1-\tau)}{P_t A_t}$ yields:

$$mc_t = w_t - p_t - a_t \qquad (14.37)$$

- Eq.(14.37) is analogous to Eq.(3.24) in classical monetary model. Thus, Eq.(14.37) is (log-linearized) labor demand curve.

- By assuming hat At=1 is applied in the steady state, Eq.(14.36) can be shown as $MC = NC(1-\tau)$. Paying attention to this and log-linearizing Eq.(14.36) yields:

$$mc_t = n_t + c_t - a_t$$

- Plugging Eqs.(14.32) and (14.35) into this yields:

$$mc_t = 2y_t - 2a_t \qquad (14.38)$$

- When price stickiness is zero, that is, prices are completely flexible, the FONC for firms implies $MC_t = (\varepsilon - 1)/\varepsilon$, that is, the marginal cost always equals to the inverse of markup.
- This means that the following is applied in flexible price equilibrium:

$$mc_t = 0 \qquad (14.39)$$

- Further, suppose that there is a relationship between the GDP and the latent GDP, namely, natural rate of output, as follows:

$$y_t = \tilde{y}_t + \bar{y}_t \qquad (14.40)$$

where $\tilde{y}_t$ denotes the GDP gap, $\bar{y}_t$ denotes the latent output or natural rate of output (both of them are logarithmic).

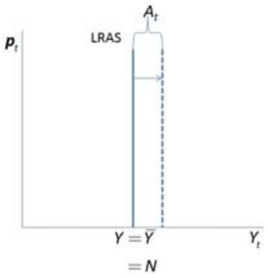
- In the flexible price equilibrium, there is no business cycle stemming from nominal rigidities. Then, the GDP gap is zero and the following is applied:

$$\tilde{y}_t = 0 \qquad (14.41)$$

- Plugging Eqs.(14.39)--(14.41) into Eq.(14.38) yields the following, which implies that fluctuation in the laten GDP just depends on the productivity:

$$\bar{y}_t = a_t \qquad (14.42)$$

Fig. 14-1: Long-run Aggregate Supply Curve and Productivity in New Keynesian Model



- In (basic) new Keynesian model, a change in the productivity changes the latent output (natural rate of output).
- That is, this change shifts the long-run aggregate supply curve.

- Plugging Eqs.(14.40) and (14.42) into Eq.(14.38) yields:

$$mc_t = 2\tilde{y}_t \qquad (14.43)$$

which implies that the GDP gap depends on the percentage deviation of the marginal cost from its steady state value.

14.4 Aggregate Demand and Aggregate Supply

- Plugging Eqs.(14.40) and (14.42) into Eq.(14.33) yields the aggregate demand equation as follows:

$$\tilde{y}_t = \mathrm{E}_t\left(\tilde{y}_{t+1}\right) - \hat{i}_t + \mathrm{E}_t\left(\pi_{t+1}\right) + \overline{r}_t \qquad (14.44)$$

where $\overline{r}_t \equiv \mathrm{E}_t\left(a_{t+1}\right) - a_t$ denotes the natural rate of interest.
- Eq.(14.44) is dubbed New Keynesian IS Curve.

- Solving Eq.(14.44) forward yields:

$$\tilde{y}_t = -\sum_{k=0}^{\infty} \mathrm{E}_t\left(\hat{r}_{t+k} - \overline{r}_{t+k}\right) \qquad (14.45)$$

where $\hat{r}_t \equiv \hat{i}_t + \mathrm{E}_t\left(\pi_{t+1}\right)$ denotes the percentage deviation of the real interest rate from its steady state value.
- Eq.(14.45) shows that the GDP gap depends on the sum of difference between the real interest rate and the natural rate of interest.

Proof of Eq.(14.45)

- Forwarding Eq.(14.44) one period and plugging recursively yields:

$$\tilde{y}_t = \mathrm{E}_t\left(y_{t+T}\right) - \mathrm{E}_t\left(\hat{r}_{t+T-1} - \overline{r}_{t+T-1}\right) - \mathrm{E}_t\left(\hat{r}_{t+T-2} - \overline{r}_{t+T-2}\right)$$
$$- \cdots - \mathrm{E}_t\left(\hat{r}_{t+1} - \overline{r}_{t+1}\right) - \left(\hat{r}_t - \overline{r}_t\right)$$

- Suppose that the far-off GDP is not affected by monetary policy and impose $\lim_{T\to\infty} \mathrm{E}_t\left(\tilde{y}_{t+T}\right) = 0$. Then, we get:

$$\tilde{y}_t = -\left(\hat{r}_t - \overline{r}_t\right) - \left(\hat{r}_{t+1} - \overline{r}_{t+1}\right) - \left(\hat{r}_{t+2} - \overline{r}_{t+2}\right) - \cdots$$
$$= -\sum_{k=0}^{\infty} \mathrm{E}_t\left(\hat{r}_{t+k} - \overline{r}_{t+k}\right)$$

which is Eq.(14.45) itself.

- Plugging Eq.(14.43) into Eq.(14.25) yields aggregate supply curve as follows:

$$\pi_t = \beta \mathrm{E}_t\left(\pi_{t+1}\right) + 2\kappa\tilde{y}_t \qquad (14.46)$$

which is dubbed New Keynesian Phillips Curve.

$$\tilde{y}_t = \mathrm{E}_t\left(\tilde{y}_{t+1}\right) - \hat{i}_t + \mathrm{E}_t\left(\pi_{t+1}\right) + \bar{r}_t \quad (14.44)$$

$$\pi_t = \beta \mathrm{E}_t\left(\pi_{t+1}\right) + 2\kappa\tilde{y}_t \quad (14.46)$$

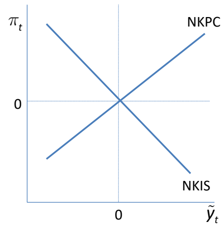
- Eq.(14.44) determines the GDP gap given exogenous natural rate of interest and the real interest rate.
- Eq.(14.46) determines the inflation given the GDP gap.
- To determine the real interest rate, the nominal interest rate must be determined earlier.
- That is, real variables are not determined unless monetary policy is given under nominal rigidities.

$$\tilde{y}_t = \mathrm{E}_t\left(\tilde{y}_{t+1}\right) - \hat{i}_t + \mathrm{E}_t\left(\pi_{t+1}\right) + \bar{r}_t \quad (14.44)$$

$$\pi_t = \beta \mathrm{E}_t\left(\pi_{t+1}\right) + 2\kappa\tilde{y}_t \quad (14.46)$$

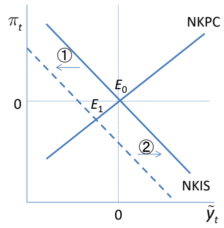
- This means that monetary policy is not neutral.
- These facts are contrast well with facts that real variables are determined independently of nominal and monetary policy is neutral variables in classical monetary model.

Fig. 14-2: NKIS and NKPC



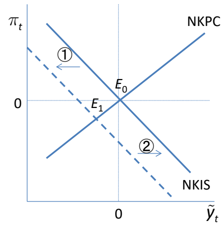
- The NKIS Eq.(14.44) and the NKPC Eq.(14.46) are shown in Fig. 14-2.

Fig. 14-3: An Increase in the Productivity



- Suppose that the productivity increases temporally.
- The NKIS shifts toward left (①). The equilibrium shifts E_0 to E_1.
- Both the inflation and the GDP gap become negative.

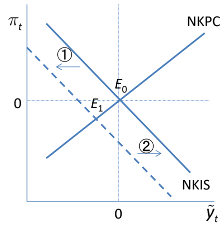
Fig. 14-3: An Increase in the Productivity



- There is a mechanism behind this.

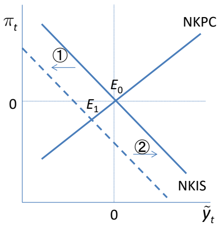
1. A decrease in the marginal cost stemming from an increase in the productivity.
2. A decrease in the optimal price chosen by firms.
3. A decrease in the inflation.
4. A decrease in the real interest rate through a decrease in the expected inflation.
5. An increase in the consumption (The income increases through an increase in the productivity).

Fig. 14-3: An Increase in the Productivity



6. Because the price does not decrease sufficiently, however, an increase in the consumption is less than an increase in the latent GDP.
7. An increase in the income is also less than an increase in the latent GDP (The employment falls).

Fig. 14-3: An Increase in the Productivity



- When the nominal interestrate lowers, however, the NKIS is shift back to right (②).
- The equilibrium moves to E_0. Both the inflation and the GDP gap return to zero.

8. Further decrease in the real interest rate stemming from a decrease in the nominal interest rate.
9. Further increase in the consumption.
10. An increase in the income (Recovery of the employment).

14.5 Monetary Policy and Determinacy

- As mentioned, real variables are not determined unless the nominal interest rate is determined earlier.
- Suppose that monetary policy is conducted following a feedback rule as follows:

$$\hat{i}_t = \varphi_\pi \pi_t + \zeta_t \quad (14.47)$$

where ζ_t denotes the monetary policy shock.

- Needless to say, Eq.(14.47) is a class of Taylor rule and is same as Eq.(3.32) in classical monetary model as long as the monetary policy shock is ignored.
- We have already shown that the price cannot be determined unless φ_π>1 is sufficed (Real variables are determined independently of nominal variables).
- Here, we discuss on determinacy on new Keynesian model.

- Plugging Eq.(14.47) into Eq.(14.44) yields:

$$\tilde{y}_t = \mathrm{E}_t(\tilde{y}_{t+1}) - \varphi_\pi \pi_t + \mathrm{E}_t(\pi_{t+1}) + \bar{r}_t - \zeta_t \quad (14.48)$$

- Rearranging Eq.(14.46) yields:

$$\tilde{y}_t = \frac{1}{2\kappa}\pi_t - \frac{\beta}{2\kappa}\mathrm{E}_t(\pi_{t+1}) \quad (14.49)$$

- Rearranging Eq.(14.48) yields:

$$\pi_t = -\frac{1}{\varphi_\pi}\tilde{y}_t + \frac{1}{\varphi_\pi}\mathrm{E}_t(y_{t+1}) + \frac{1}{\varphi_\pi}\mathrm{E}_t(\pi_{t+1}) + \frac{1}{\varphi_\pi}\bar{r}_t - \frac{1}{\varphi_\pi}\zeta_t \quad (14.50)$$

- Plugging Eq.(14.49) into Eq.(14.48) yields:

$$\pi_t = \frac{2\kappa}{1+2\kappa\varphi_\pi}\mathrm{E}_t(y_{t+1}) + \frac{2\kappa\beta}{1+2\kappa\varphi_\pi}\mathrm{E}_t(\pi_{t+1}) + \frac{2\kappa}{1+2\kappa\varphi_\pi}\bar{r}_t - \frac{2\kappa}{1+2\kappa\varphi_\pi}\zeta_t \quad (14.51)$$

- Pluggging Eq.(14.50) into Eq.(14.46)yields:

$$\tilde{y}_t = \frac{1}{1+2\varphi_\pi\kappa}\mathrm{E}_t(y_{t+1}) + \frac{1-\varphi_\pi\beta}{1+2\varphi_\pi\kappa}\mathrm{E}_t(\pi_{t+1}) + \frac{1}{1+2\varphi_\pi\kappa}\bar{r}_t - \frac{1}{1+2\varphi_\pi\kappa}\zeta_t \quad (14.52)$$

- Eq.(14.51) is the NKIS in which NKPC is plugged while Eq.(14.52) is the NKPC in which the NKIS is plugged.
- Eqs.(14.51) and (14.52) can be expresssesd as:

$$\begin{bmatrix}\tilde{y}_t \\ \pi_t\end{bmatrix} = \mathrm{M}\begin{bmatrix}\mathrm{E}_t(\tilde{y}_{t+1}) \\ \mathrm{E}_t(\pi_{t+1})\end{bmatrix} + \mathrm{N}\begin{bmatrix}\bar{r}_t \\ \zeta_t\end{bmatrix}$$

where:

$$\mathrm{M} \equiv \begin{bmatrix}\frac{1}{1+2\phi_\pi\kappa} & \frac{1-\phi_\pi\beta}{1+2\phi_\pi\kappa} \\ \frac{2\kappa}{1+2\phi_\pi\kappa} & \frac{2\kappa+\beta}{1+2\phi_\pi\kappa}\end{bmatrix} \text{ and } N \equiv \begin{bmatrix}\frac{1}{1+2\phi_\pi\kappa} & -\frac{1}{1+2\phi_\pi\kappa} \\ \frac{2\kappa}{1+2\phi_\pi\kappa} & -\frac{2\kappa}{1+2\phi_\pi\kappa}\end{bmatrix}.$$

- The necessary and sufficient condition for that Eq.(14.52) has unique solution is that the number of non-predetermined endogenous variable equals to the number of roots in a unit circle (Blanchard and Kahn, 1980).
- This condition is sufficed when the followings are sufficed:

$$|\det(\mathrm{M})| < 1 \quad (14.53)$$

$$|-\mathrm{tr}(\mathrm{M})| < 1 + \det(\mathrm{M}) \quad (14.54)$$

where:

$$\det(\mathrm{M}) = \frac{1}{1+2\phi_\pi\kappa}\frac{2\kappa+\beta}{1+2\phi_\pi\kappa} - \frac{1-\phi_\pi\beta}{1+2\phi_\pi\kappa}\frac{2\kappa}{1+2\phi_\pi\kappa}$$

$$= \frac{\beta}{1+2\phi_\pi\kappa} \quad (14.55)$$

$$\mathrm{tr}(\mathrm{M}) = \frac{1+2\kappa+\beta}{1+2\phi_\pi\kappa} \quad (14.56)$$

- First, we consider Eq.(14.53). Combining Eqs.(14.53) and (14.55) yields:

$$\beta < 1 + 2\varphi_\pi\kappa$$

- Here, because of $\varphi_\pi \geq 0$, $\kappa > 0$ and $\beta \in (0,1)$, absolute-value sign is removed. Eq.(14.53) is definitely sufficed because of this equality.
- Next, we consider Eq.(14.54). Combining Eqs.(14.54)--(14.56) yields:

$$\frac{1+2\kappa+\beta}{1+2\varphi_\pi\kappa} < 1 + \frac{\beta}{1+2\varphi_\pi\kappa}$$

- Absolute-value sign is removed, too. This can be rewritten as:

$$\varphi_\pi > 1 \quad (14.57)$$

- In classical monetary model, Taylor rule which suffices $\varphi_\pi > 1$ determines the inflation uniquely.
- Similar to one in classical monetary model, Eq.(14.57) is a condition for uniqueness.
- In new Keynesian model, monetary policy is not neutral and not only nominal variables, but also real variables are determined dependently of monetary policy.
- Further, iff Eq.(14.57), namely, $\varphi_\pi > 1$ is sufficed, both nominal and real variables are determined uniquely.
- Taylor principle is $\varphi_\pi > 1$ in both clasical monetary and new Keynesian models. Although $\varphi_\pi > 1$ is a condition for determining just nominal variables in classical monetary model, however, this is a condition for not only determining nominal but also real variables in new Keynesian model.

- Next, we consider exogenous money supply rule. In classical monetary model, exogenous money supply rule determines nominal variables uniquely.
- Plugging Eqs.(14.40) and (14.42) into Eq.(14.13) yields:

$$l_t = \tilde{y}_t - \eta \hat{i}_t + a_t \tag{14.58}$$

where $l_t \equiv m_t - p_t$ denotes the percentage deviation of the real money balance from its steady stae value.
- Plugging Eq.(14.58) into Eq.(14.44) yields:

$$(1+\eta)\tilde{y}_t = \eta \mathrm{E}_t(y_{t+1}) + l_t + \eta \mathrm{E}_t(\pi_{t+1}) + \eta \overline{r}_t - a_t \tag{14.59}$$

- By taking first differencial on the definition of the real money balance, we get:

$$l_t - l_{t-1} = -\pi_t + \Delta m_t \tag{14.60}$$

- Eqs.(14.49), (14.59) and (14.60) can be expressed as:

$$\mathrm{M}_{M,0}\begin{bmatrix}\tilde{y}_t\\ \pi_t\\ l_{t-1}\end{bmatrix} = \mathrm{M}_{M,1}\begin{bmatrix}\mathrm{E}_t(\tilde{y}_{t+1})\\ \mathrm{E}_t(\pi_{t+1})\\ l_t\end{bmatrix} + N_M\begin{bmatrix}\overline{r}_t\\ a_t\\ \Delta m_t\end{bmatrix}$$

- By calculating the inverse of $\mathrm{M}_{M,0}$, namely, $\mathrm{M}_{M,0}^{-1}$, we have:

$$\begin{bmatrix}\tilde{y}_t\\ \pi_t\\ l_{t-1}\end{bmatrix} = \mathrm{M}_M\begin{bmatrix}\mathrm{E}_t(\tilde{y}_{t+1})\\ \mathrm{E}_t(\pi_{t+1})\\ l_t\end{bmatrix} + \mathrm{M}_{M,0}^{-1}\mathrm{N}_M\begin{bmatrix}\overline{r}_t\\ a_t\\ \Delta m_t\end{bmatrix} \tag{14.61}$$

with $\mathrm{M}_\mathrm{M} \equiv \mathrm{M}_{M,0}^{-1}\mathrm{M}_{M,1}$.
- Eq.(14.61) includes one predetermined and two non-predetermined variables. Numerical analysis shows that 2 of roots in MM are inside unit circle while one of them is outside or on unit circle (Gali, 2008).

- Thus, solution has always uniqueness under money supply rule.
- This result is analogous to one in classical monetary model under exogenous money supply rule where nominal variables are determined uniquely.

14.6 Numerical Analysis

- In new Keynesian model, it is absolutely impossible to obtain exprecit analytical solution, different from classical monetary model.
- The, we calculate dynamics by numerical analysis.
- To calculate, we have to decide paths of exogenous shocks. Here, we decides paths as follows:

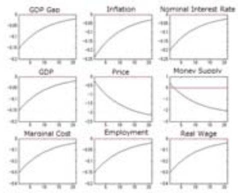
$$\begin{aligned} a_t &= \rho a_{t-1} + v_t \\ \zeta_t &= \rho \zeta_{t-1} + \xi_t \\ \Delta m_t &= \rho \Delta m_{t-1} + \upsilon_t \end{aligned} \tag{14.62}$$

where v_t, ξ_t and υ_t are i.i.d. and shocks whose expected value is zero.
- Such kind of processes are dubbed first-order auto-regressive processes or AR(1) processes.

- We have to set parameter values to analyze numerically.
- Here, we set the subjective discount factor β to 0.99, the price stickiness θ to 0.75, the semi-elasticity of real money balance to nominal interest rate η to 4 and coefficients of AR(1) processes ρ to 0.9.
- The timing of the model is quarterly, that is, one period corresponds to three month.
- Thus, our parameterization implies that the nominal interest rate in the steady state is $1/0.99 \times 4 \simeq 4\%$, the duration of price revision is は1/(1－0.75)＝4 quarters which corresponds to Taylor (1999).
- Under Taylor rule, we set $\varphi_\pi = 1.5$. This implies that the nomina interest rate is hiked 1.5% to 1% increase in the inflation.

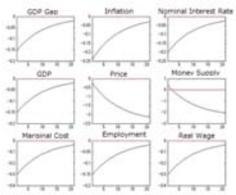
- The model which consists of equations such as Eqs.(14.44), (14.46) and (14.47) or (14.58), and (14.60) is dubbed Rational Expectation Macroeconomic Model.
- The method to solve the model is found by Blanchard and Kahn (1980) and Soderlind, Klein and Uhlig found easier ways to solve.
- Here, we adopt a solver, Dynare based on Michel Juliard’s algorythm and working on MATLAB.
- Behavior on dynamics of variables, solved by the method is Impulse Response.

Fig. 14-2: IRFs to Monetary Policy Shock under Taylor Rule



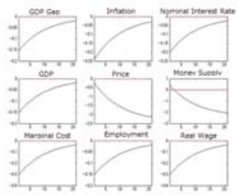
- Fig. 14-2 shows IRFs to monetary policy shock which increases the nominal interest rate 1% under Taylor rule.
- The vertical axis is the percentage deviation from the steady state and the horizontal axis is time after the shock. The timing is quarterly and the shock arises in period 0.

Fig. 14-2: IRFs to Monetary Policy Shock under Taylor Rule



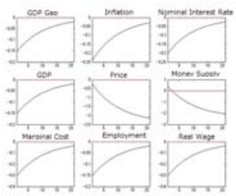
- The nominal interest rate is about to increase drastically in period 0 and the GDP gap becomes negative through a pressure on the NKIS.
- A decrease in the GDP gap decrease the inflation.

Fig. 14-2: IRFs to Monetary Policy Shock under Taylor Rule



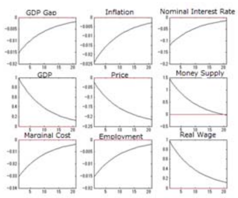
- The nominal interest rate lowers through Taylor rule.
- A pressure which decreases the nominal interest rate stemming from the monetary policy shock decreases consumption through Eq.(14.12).
- This decrease in the demand for goods decreases the marginal cost through Eq.(14.36).
- That is, firms choose the optimal price $\tilde{p}_t$ lower than it in previous period.

Fig. 14-2: IRFs to Monetary Policy Shock under Taylor Rule



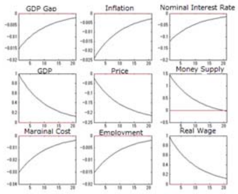
- Because of this, as shown in Eq.(14.16), the inflation decreases and the price decreases.
- Now, because the productivity does not change, latent GDP does not change as shown in Eq.(14.42).
- The, GDP gap equals to GDP.
- Thus, the employment decreases.
- Since the productivity does not change, a decrease in the marginal cost decreases the real wage immediately.

Fig. 14-3: IRFs to Productivity Shock under Taylor Rule



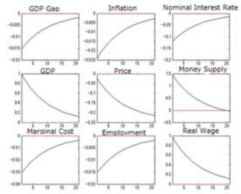
- Fig. 14-3 shows IRFs to productivity shock which increases productivity 1% under Taylor rule.
- An increase in the productivity increases the GDP gap through the NKIS.
- A dcrease in the GDP gap decreases the inflation through the NKPC.
- The nominal interest rate decreases more than the inflation because of Taylor rule.

Fig. 14-2: IRFs to Monetary Policy Shock under Taylor Rule



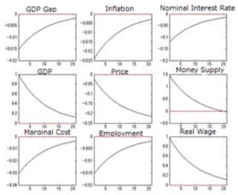
- As shown in Eq.(14.58), the money supply increases because of a decrease in the nominal interest rate.
- The money supply, however, becomes negative after 3 quarters because both of the GDP gap and the price decrease.

Fig. 14-3: IRFs to Productivity Shock under Taylor Rule



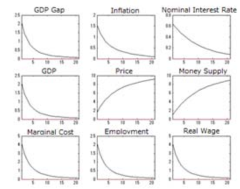
- As shown in log-linearized Eq.(14.36), an increase in the productivity decreases the marginal cost.
- Thus, firms choose optimal price $\tilde{p}_t$ lower than previous period. Then, both the inflation and the price fall.
- Because a decrease in the price is sluggish, the real wage increases.
- An increase in the productivity increases latent GDP. Further, GDP rises.

Fig. 14-3: IRFs to Productivity Shock under Taylor Rule



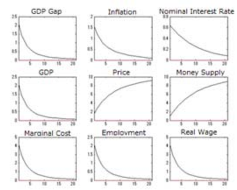
- An increase in the GDP induces an increase in the consumption, as shown in Eq.(14.32).
- Log-linearized Eq.(14.36) implies that a decreases in the marginal cost through an increase in the productivity and an increase in the consumption decreases employment.
- Much employment is no longer needed when the productivity increases.
- Thus, the employment decreases. Then, the GDP gap decreases.

Fig. 14-4: IRFs to Money Supply Shock under Money Supply Rule



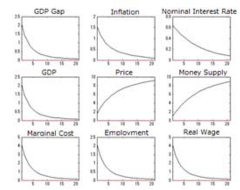
- Fig. 14-4 shows IRFS to the money supply shock which increases money supply growth 1% under the money supply rule.
- While the money supply grows sustainably, the growth rate gradually decreases because the growth rate follows AR(1).
- An increase in the money supply applies a buildup pressure to the real money balance, as shown in Eqs.(14.58) and (14.60)

Fig. 14-4: IRFs to Money Supply Shock under Money Supply Rule



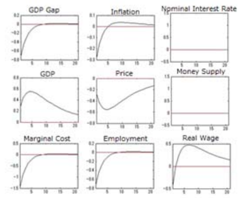
- This buildup pressure increases the GDP. Now, the GDP gap rises because the latent GDP does not cahnge.
- An increase in the GDP gap induces an increase in the marginal cost simultaneously because firms choose the optimal price $\tilde{p}_t$ higher than it in previous period, reflecting an increase in the demand for goods.
- Then, both the inflation and the price increase through Eq.(14.16).

Fig. 14-4: IRFs to Money Supply Shock under Money Supply Rule



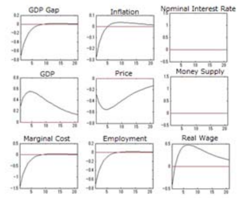
- Because an increase in the price exceeds an increases the money supply, the real money balance decreases.
- Meanwhile, GDP increases. As a result, the nomilal interest rate increases.
- An increase in the GDP increases employment. In addition, the real wage increases through an increase in the marginal cost because the productivity does not change.

Fig. 14-5: IRFs to Productivity Shock under Money Supply Rule



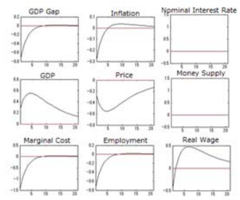
- Fig. 14-5 shows IRFs to the productivity shock which increases productivity 1% under the money supply rule.
- The money supply is determined exogenously and is constant because it is not affected by changes in the productivity.
- Similar to Taylor rule, an increase in the productivity decreases the marginal cost and the GDP gap.

Fig. 14-5: IRFs to Productivity Shock under Money Supply Rule



- On the other side, the GDP increases because the productivity increases.
- A decrease in the marginal cost decreases the inflation immediately. Thus, the price falls.
- The real wage decreases with a decrease in the marginal cost. The real wage, however, becomes positive after 3 quarters with an increase in consumption.

Fig. 14-5: IRFs to Productivity Shock under Money Supply Rule



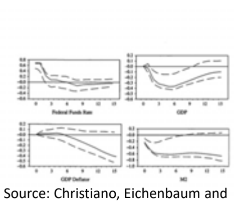
- As shown in Eq.(14.58), an increase in the productivity makes the real money balance ascending.
- Because the money supply does not change and the price decreases, the real money balance rises.
- On the other hand, the GDP rises and an increase in the GDP cancels a pressure stemming from an increase in the real money balance. Thus, the nominal interest rate does not change.
- The demand for labor falls and the employment decreases.

- These numerical analysis gives some contribution.

1. Showing how new Keynesian model works.
2. Calibration shows the effect of shocks is whether consistent or not with data.
3. Showing tat the monetary policy has an important role to decide both nominal and real variables.

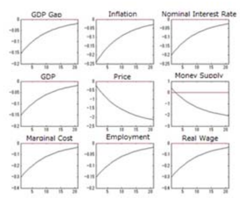
- Now, we investigate '2' roughly.

Fig. 10-1: Estimated Dynamic Response to a Monetary Policy Shock



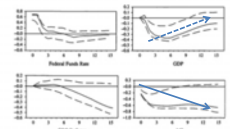
Source: Christiano, Eichenbaum and Evans (1999)

Fig. 14-2: IRFs to Monetary Policy Shock under Taylor Rule



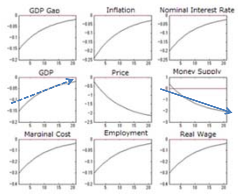
- Christiano, Eichenbaum and Evans (1999) assume more complicate Taylor rule, different from Eq.(14.47). Thus we can not compare our calibration with data simply.
- In addition, our calibration accompanies with strong assumption on parameterization. Just the price stickiness and the semi-elasticity of real money demand to nominal interest rate freely.
- For example, the labor supply elasticity, which is the elasticity of hours of labor to rate of real wage, namely, Frisch elasticity is assumed 1 implicitly.
- However, consistency with data is better than classical monetary model.

Fig. 10-1: Estimated Dynamic Response to a Monetary Policy Shock



Source: Christiano, Eichenbaum and Evans (1999)

Fig. 14-2: IRFs to Monetary Policy Shock under Taylor Rule



14.7 Criticisms for New Keynesian Model

- There are some criticisms for new Keynesian model.
- New Keynesian model is a model which is classical monetary model with nominal rigidity. Thus, feature on new Keynesian model appears on the NKPC stemming from the FONC for firms. Thus, it can be said that those criticisms fall on the NKPC.
- Many criticisms are related to consistency with data.
- On the other hand, research which makes model's prediction more consistent with data.

14.8 Evolving New Keynesian Model

- New Keynesian model is extending.

1. Optimal Monetary Policy
2. Determinacy on Equilibrium
3. Rules vs Discretion
4. Sticky Wage
5. Open Economy

and so on.